

The following article was written in May 2004 on the occasion of Professor Simonov's 70-th anniversary. I intended to publish it in the Yuri Simonov Festschrift. For reasons beyond my control it could not be included in this Festschrift. The first part (written in Russian) presents a personal recollection. The second part (in English) is scientific, it gives a mini-review of central charges in the Seiberg-Witten model, with a few new observations.

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Minneapolis

ПОЗДРАВЛЕНИЕ ИЗ ПРОСТРАНСТВЕННО-ВРЕМЕННОГО ДАЛЕКА ...

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В 1984 году Юрию Антоновичу исполнилось 50 лет. Кажется, был юбилейный семинар, в актовом зале. Как сейчас вижу – Юрий Антонович вверху, на подиуме, как обычно излучающий спокойствие и уверенность, и этот царственный жест, неуловимое прикосновение руки к чуть сбитой пряди роскошных черных волос, с едва намеченным серебрянным узором, и вот прядь уже на месте, там где ей и положено быть... Помню как щедро раздавал Юрий Антонович, направо и налево, каждому немножко сияния высших сфер.

В тот день в кабинете который я делил с Аллой Михайловной, Поповым и Переломовым, и куда после семинара пришел Юрий Антонович, состоялся большой разговор о науке. Это было время больших ожиданий. Квантовая хромодинамика еще на подъеме. Грибовские копии, инстантонные модели вакуума, глюболы и гибриды — при звуке этих магических заклинаний у многих (в нашем маленьком мирке) закипала кровь. Казалось, вот еще последнее усилие, и квантовая хромодинамика будет решена. Это было до суперсимметричной парадигмы, до струн и D -бран.

Наш следующий подобный разговор произошел всего пару лет спустя, в начале перестройки, в курортном поселке Салацгрива. Летом здесь собиралась элита московской и ленинградской интеллигенции. Для нас – а я приехал с Юлей и Аней на полдня, по дороге из Пярну в Ригу – тогда это был немыслимый запад. Я помню чинные обмены приветствиями с друзьями Юрия Антоновича и Аллы Михалойвны и долгую прогулку по дорожке в сосновом лесу, вдоль берега моря, гомон птиц и нежаркое балтийское солнце... Первая тема, как всегда, политика. Всерьез ли открылось окно в цивилизацию, не закроется ли? Горбачев ... Лигачев ... (кажется, Егор Кузмич, да кто помнит его сегодня) ... Разве могли мы тогда гадать, что всего несколько лет спустя Салацгрива станет частью независимой Латвии, а Советский Союз – “оплот всего прогрессивного человечества” – рассеется как дурной сон, оставив лишь несчастья, нищету и горькие воспоминания миллионам ни в чем не повинных людей?

Ну а потом – тоже не новость – плавно перетекли в физику. Уже нарастал вал струнной тематики. Первая струнная революция о необходимости которой все время твердили ... свершилась. Джон Эллис, приехавший в ИТЭФ на пару недель, каждый день звонил в ЦЕРН¹ узнать не завершилось ли – не дай бог в его отсутствие – построение “теории всего”. Свежую идею, как это всегда бывает – и как и должно быть – несли в массы самые молодые и яркие. Да здравствуют пионеры! Суперструны (и суперсимметрия в целом) из затерянных и экзотических углов теории врывались в mainstream, неудержимо оттесняя в сторону предыдущий mainstream – квантовую хромодинамику. О, это пьянящее чувство прорыва ...

Все это обсуждали мы тогда с Юрием Антоновичем. Наши точки зрения отчасти совпадали, но во многом и расходились. У каждого был свой сценарий. Сейчас, почти двадцать лет спустя, вспоминая свои предсказания, я смеюсь... Увы, в науке как и в общественной жизни, будущее, как правило, оказывается значительно более неожиданным, чем любые ожидания.

Разговор этот мы так и не окончили – дорожка в лесу оказалась короче разговора – да и можно ли его закончить вообще? За двадцать лет многое изменилось, и в теории и в теоретическом ИТЭФ. Иных уж нет, а те далече ... Минимальная суперсимметричная стандартная модель выродилась в прогонку десятка, если не сотни, параметров. Квантовая хромодинамика так и не решена, хотя кое-какие аппетитные куски откушены. Революционный задор теории струн как-то сник. “Теории всего” кажется не получилось. А ведь были еще и вторая и третья струнные революции.²

Характер физики высоких энергий меняется на глазах – связь с эмпирическими корнями слабеет, уклон в сторону математической физики растет, и с этим уже ничего не поделаешь. Правда, выросла и возмужала astroparticle physics (long live dark matter and dark energy, the basis of our universe! – or ... is it multiverse?..), и возродилась в новом облике старая идея Калуцы и Клайна – в виде больших дополнительных измерений –

¹Для молодых людей спешу добавить, что позвонить за границу из ИТЭФа тогда можно было только с одного телефона – у Помелова на столе в международном отделе, и прямого набора, конечно, не существовало. Разговор надо было заказывать заранее, через оператора. Джон наверняка материл про себя всю эту тягомотину, и тем не менее процедура повторялась изо дня в день.

²Обычно в струнной литературе подразумевается, что в этом контексте слово “революция” – синоним высочайшего достижения, но в силу понятных исторических причин я все равно каждый раз вздрагиваю.

с которой соскучиться невозможно. Что это: относительно небольшой зигзаг на историческом пути физики, или поворот всерьез и надолго?

Долой, долой ностальгию по прошлому. Лучшее лекарство – хорошая шутка. Вот недавно набрел в интернете на миниатюру так искусно сработанную, что поначалу принял за чистую монету, лишь позже обратив внимание на день публикации – первое апреля. И так, April 1, 2004; posted by Peter Woit (Department of Mathematics, Columbia University).

String Theorists Finally Admit Defeat³

The news that next week's "Science Times" will run an article by *New York Times* reporter James Glanz in which several leading string theorists say that they are giving up on the idea is rapidly spreading throughout the particle theory community. Evidently Glanz recently went down to Princeton to interview Edward Witten, who took the opportunity to announce that he has changed his mind about whether string theory will ever be a "Theory of Everything". When Glanz contacted other string theorists and read to them what Witten had said, almost all of them told him that they too had been having their doubts about the theory.

Glanz quotes Witten as follows:

"One night a few weeks ago I was sitting at my kitchen table trying to make sense of Douglas's latest work on the Kachru-Kalosh-Linde-Trivedi (KKLT) proposal and all of a sudden it really hit me that this is a completely lost cause. If perturbative string theory has any relation to Planck scale physics, then KKLT or something like it should work and string theory is vacuous since it can never predict anything. If perturbative string theory isn't useful then we really don't have anything since we've never been able to come up with a non-perturbative version that makes sense. Twenty years of this is enough. It's time to give up."

When Glanz asked him what he intends to do now, Witten responded:

"I don't really know. There are still promising ideas about using string theory to solve QCD, and I could keep working on those. Maybe I should take up something completely different, like biology. I'm starting to worry that John Horgan was right about the 'End of Science'. Right now I just definitely need a long vacation."

When Glanz read Witten's statement over the phone to David Gross, Frederick W. Gluck Professor of Physics at UCSB and Director of the Fred Kavli Institute for Theoretical Physics, Gross thought for a moment and then told him "Yeah, despite my quote last year from Churchill, I've also been thinking of giving up. Not sure though how I'm going to break this to the two Feds."

The news of Glanz's article has had dramatic effects at many universities and research institutes. At MIT yesterday, Professor Barton Zwiebach shocked students in his Physics 8.251 "String Theory for Undergraduates" class by announcing that

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he wasn't going to collect the homework due that day and was canceling his lectures for the rest of the semester. He also asked Cambridge University Press to halt publication of his new undergraduate textbook called "A First Course in String Theory", the release of which had been planned for next month.

Search committees at several institutions that hadn't finished their hiring yet this season held new meetings to decide how to react to the news. A prominent theorist at a University of California campus told me in an e-mail that "our chair had the phone in his hand and had already dialed the number of a string theory graduate student from Princeton we were going to offer a post-doc to. I ran into his office as soon as I heard the news and stopped him just in time. Last week we were sure that string theorists were the smartest guys around and considered only them for jobs, but now there's no way we're going to hire any more, ever!"

At the Institute in Princeton this year's "Summer Program for Graduate Students in String Theory" scheduled for July has been canceled, with one of its organizers remarking "what graduate student would now be crazy enough to show up for a program like this?" Next week's conference on "The Status of M-theory" at the Michigan Center for Theoretical Physics has also been canceled on very short notice. The director there, Michael Duff, commented "We had to do this because the status of M-theory is all too clear. It's passed on! This theory is no more! It has ceased to be! It's expired and gone to meet its maker! ... This is an ex-theory!"

А вот еще одна шутка, которая, как мне кажется, забавна тем, что дает представление об отношении современных студентов к разным областям физики – из тех, что нынче входят в "джентельменский" набор НЕР-теоретика (так же как и о самой композиции этого джентельменского набора):

Physical Theories as Women

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0. Newtonian gravity is your high-school girlfriend. As your first encounter with physics, she's amazing. You will never forget Newtonian gravity, even if you're not in touch very much anymore.

1. Electrodynamics is your college girlfriend. Pretty complex, you probably won't date long enough to really understand her.

2. Special relativity is the girl you meet at the dorm party while you're dating electrodynamics. You make out. It's not really cheating because it's not like you call her back. But you have a sneaking suspicion she knows electrodynamics and told her everything.

3. Quantum mechanics is the girl you meet at the poetry reading. Everyone thinks she's really interesting and people you don't know are obsessed about her.

You go out. It turns out that she's pretty complicated and has some issues. Later, after you've broken up, you wonder if her aura of mystery is actually just confusion.

4. General relativity is your high-school girlfriend all grown up. Man, she is amazing. You sort of regret not keeping in touch. She hates quantum mechanics for obscure reasons.

5. Quantum field theory is from overseas, but she doesn't really have an accent. You fall deeply in love, but she treats you horribly. You are pretty sure she's fooling around with half of your friends, but you don't care. You know it will end badly.

6. Cosmology is the girl that doesn't really date, but has lots of hot friends. Some people date cosmology just to hang out with her friends.

7. Analytical classical mechanics is a bit older, and knows stuff you don't.

8. String theory is off in her own little world. She is either profound or insane. If you start dating, you never see your friends anymore. It's just string theory, 24/7.

Ну, а если более серьезно, на всякое явление можно взглянуть двояко: “стакан либо наполовину пуст, либо наполовину полон...” Люди не боги – зачем нам теория всего? Так интереснее. Вопросов много, и на последнюю страницу за ответом не заглянешь. В общем, сегодня, как и всегда,

“... на том рубеже,
крутом вираже,
на узкой меже меж еще и уже ...”

Так стоит ли брюзжать: “вот в наше время...”? (Это я, конечно, сам себе говорю...)

Рассуждая логически, я должен был бы написать в этот сборник о квантовой хромодинамике, теме центральной для Юрия Антоновича на протяжении последних двадцати пяти лет. Стохастическая модель вакуума и ее приложения в низкоэнергетической адронной физике – любимое дитя Юрия Антоновича и Гюнтера Доша. Физика адронов – и моя первая любовь, которая, конечно же, не забывается. В последний раз я вернулся к ней всерьез и надолго, в середине 90-х годов прошлого века (рука не поворачивается, но придется написать, – прошлого века). Взявшись дружно с Колей Уральцевым и Аркадием Вайнштейном, удалось закончить разложение по тяжелым кваркам, основы которого были заложены Мишей Волошиным и мной в 1980-х. Потом, в 1999-ом около полугода пытался хоть как-то продвинуться в кварк-адронной дуальности. С практической точки зрения вопрос чрезвычайно важный, да как к нему подступиться? В общем, продвинулся далеко не так далеко как хотелось.

С тех пор были встречи, но какие-то мимолетные. Интересы мои в заметной мере сместились. Хотя и надеюсь вернуться в ближайшем

будущем к адронным k -струнам и следствиям планарной эквивалентности, но писать надо о том, что интересует в данный момент. Поэтому – да простит меня Юрий Антонович – напишу о суперсимметрии, точнее, об одном ее аспекте – центральных зарядах. Тема эта имеет прямое отношение к ИТЭФу. Сам того не подозревая, у истоков ее стоял Женя Богомольный, в то время аспирант ИТЭФ. Хорошо помню, работа писалась на голубятне, где и сам я тогда обретался. То что сегодня называется Bogomolny limit, Bogomolny completion, Bogomol’nyi-Prasad-Sommerfield (BPS) saturation⁴ [2, 3] – все это было введено Женей в досуперсимметричную эпоху, в 1974 году, в процессе разбирательства с поляковскими монополями, в котором на ранних этапах участвовали также Миша Маринов [1] и, особенно, Аркадий Вайнштейн, который, как всегда, конечно, ничего не написал. Работа Богомольного – одна из самых цитируемых из сделанных в ИТЭФе за всю историю его существования. Женя не был москвичом, и требовались специальные усилия, чтобы оставить его в ИТЭФе, каковые предприняты не были. Препринт вышел в Черноголовке.

Следующий ниже материал – что-то среднее между кратким обзором, наброском незавершенной статьи и главой ненаписанной книги.⁵ Будет ли она написана? Кто знает...

⁴The above notions are among basic entries of modern internet-based encyclopedias. For instance, ENCYCLOPEDIA 4U.com defines *Bogomol’nyi-Prasad-Sommerfield bound* as follows: “The Bogomol’nyi-Prasad-Sommerfield bound refers to a series of inequalities for solutions of partial differential equations depending on the homotopy class of the solution at infinity. This set of inequalities is very useful for solving soliton equations. Often, by insisting that the bound is satisfied (called ‘saturation’), one can come up with a simpler set of partial differential equations to solve.”

⁵К сожалению, помимо чисто литературных огрехов, не хватает времени выверить все минусы и двойки в коэффициентах. Общая структура и заключительные выводы от этого кажутся не страдают.

Remarks on Central Charges in Superalgebras

1. CENTRAL CHARGES IN SUPERALGEBRAS – GENERALITIES

In this Section we will briefly review general issues related to central charges (CC) in superalgebras.

1.1. **History.** The first superalgebra in four dimensional field theory was derived by Golfand and Likhtman [4] in the form

$$\{\bar{Q}_\alpha Q_\beta\} = 2P_\mu (\gamma^\mu)_{\alpha\beta}, \quad \{\bar{Q}_\alpha \bar{Q}_\beta\} = \{Q_\alpha Q_\beta\} = 0, \quad (1)$$

i.e. with no central charges. Possible occurrence of CC (elements of superalgebra commuting with all other operators) was first mentioned in an unpublished paper of Lopuszanski and M. Sohnius [5] where the last two anticommutators were modified as

$$\{Q_\alpha^I Q_\beta^G\} = Z_{\alpha\beta}^{IG}. \quad (2)$$

A more complete description of superalgebras with CC in quantum field theory was worked out in [6]. The only central charges analyzed in this paper were Lorentz scalars (in four dimensions). Thus, by construction, they could be relevant only to extended supersymmetries. Then, a few years later, Witten and Olive [7] showed that in supersymmetric theories with solitons, central extension of superalgebras is typical; topological quantum numbers play the role of central charges. It was generally understood that superalgebras with (Lorentz-scalar) central charges can be obtained from superalgebras without central charges in higher-dimensional space-times by interpreting some of the extra components of the momentum as CC's (see e.g. [8]). That not all CC's are of this type was known at this time at the algebraic level (see e.g. [12]), but the dynamical role of these additional tensorial charges was not fully appreciated until somewhat later. Central charges that are antisymmetric tensors in various dimensions were introduced (in the supergravity context, in the presence of p -branes) in Ref. [13] (see also [14]). These CC's are relevant to extended objects of the domain wall type. Their occurrence in four-dimensional super-Yang-Mills theory (as a quantum anomaly) was first observed in [10]. A general theory of central extensions of superalgebras in three and four dimensions was discussed in Ref. [11]. It is worth noting that discussion in [11] of those central charges that have the Lorentz structure of P_μ was not carried out in full.

1.2. **Minimal SUSY.** The minimal number of supercharges in various dimensions is given in Table 1. Two-dimensional theories with a single supercharge, although algebraically possible, require the loss of F and $(-1)^F$. Therefore, if one wants to keep the distinction between the "bosons" and "fermions," the minimal number of supercharges in $D = 2$ is two.

D	2	3	4	5	6	7	8	9	10
ν_Q	(1*) 2	2	4	8	8	8	16	16	16
$\text{Dim}(\psi)_C$	2	2	4	4	8	8	16	16	32
# cond.	2	1	1	0	1	1	1	1	2

TABLE 1. The minimal number of supercharges, dimension of the spinorial representation and the number of additional conditions (i.e. the Majorana and/or Weyl conditions).

The minimal number of supercharges in Table 1 is given for a real representation. Then, it is clear that, generally speaking, the maximal possible number of CC's is determined by the dimension of the symmetric matrix $\{Q_i Q_j\}$ of the size $\nu_Q \times \nu_Q$, namely,

$$\nu_{\text{CC}} = \frac{\nu_Q(\nu_Q + 1)}{2}. \quad (3)$$

In fact, D anticommutators have the Lorentz structure of the energy-momentum operator P_μ . Therefore, up to D central charges could be absorbed in P_μ , generally speaking. In particular situations this number can be smaller, since although algebraically the corresponding CC's have the same structure as P_μ , they are dynamically distinguishable. The point is that P_μ is uniquely defined through the conserved and symmetric energy-momentum tensor of the theory.

The total set of CC's can be arranged by classifying CC's with respect to their Lorentz structure. Below we will present this classification for $D = 2, 3$ and 4.

1.3. $D = 2$. Consider two-dimensional theories with two supercharges. From the discussion above, on purely algebraic grounds, three CC's are possible: one Lorentz-scalar and a two-component vector,

$$\{Q_\alpha, Q_\beta\} = 2(\gamma^\mu \gamma^0)_{\alpha\beta}(P_\mu + Z_\mu) + i(\gamma^5)_{\alpha\beta} Z, \quad (4)$$

The latter case would require existence of a vector order parameter taking distinct values in different vacua. This will break Lorentz invariance and supersymmetry of the vacuum state. Limiting ourselves to supersymmetric vacua we conclude that only one (real) Lorentz-scalar central charge Z is possible. This central charge is relevant to kinks in $\mathcal{N} = 1$ theories.

1.4. $D = 3$. The central charge allowed in this case is a Lorentz-vector Z_μ , i.e.

$$\{Q_\alpha, Q_\beta\} = 2(\gamma^\mu \gamma^0)_{\alpha\beta}(P_\mu + Z_\mu), \quad (5)$$

which we should arrange Z_μ to be orthogonal to P_μ . By an appropriate choice of reference frame it can always be cast in the form $(0, 0, 1)$. In fact, this is the central

charge of the previous section elevated by one dimension. It is associated with a domain wall (or string) oriented along the second axis.

1.5. $D = 4$. Maximally one can have 10 CC's which are decomposed into Lorentz representations as $(0,1) + (1,0) + (1/2, 1/2)$:

$$\begin{aligned} \{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} &= 2(\gamma^\mu)_{\alpha\dot{\alpha}}(P_\mu + Z_\mu), \\ \{Q_\alpha, Q_\beta\} &= (\Sigma^{\mu\nu})_{\alpha\beta} Z_{[\mu\nu]}, \end{aligned} \tag{6}$$

$$\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = (\bar{\Sigma}^{\mu\nu})_{\dot{\alpha}\dot{\beta}} \bar{Z}_{[\mu\nu]}, \tag{7}$$

where $(\Sigma^{\mu\nu})_{\alpha\beta} = (\sigma^\mu)_{\alpha\dot{\alpha}}(\bar{\sigma}^\nu)_{\dot{\beta}}^\alpha$ is a chiral version of $\sigma^{\mu\nu}$ (see e.g. [19]). The anti-symmetric tensors $Z_{[\mu\nu]}$ and $\bar{Z}_{[\mu\nu]}$ are associated with domain walls, and reduce to a complex number and a spatial vector orthogonal to the domain wall. The $(1/2, 1/2)$ CC Z_μ is a Lorentz vector orthogonal to P_μ . It is associated with strings (flux tubes), and reduces to one real number and a three-dimensional unit spatial vector parallel to the string.

1.6. **Extended SUSY.** We will limit our attention here to exploring the reduction of the minimal SUSY algebra in $D = 4$ to $D = 2$ and 3, namely the $\mathcal{N} = 2$ SUSY algebra in those dimensions. As should be clear from the discussion above, the maximal number of CC's is of course the same, and the only distinction we must make is to provide a decomposition into both Lorentz and R -symmetry irreps.

- $\mathcal{N} = 2$ in $D = 3$

The superalgebra can be decomposed into Lorentz and R -symmetry tensorial structures as follows:

$$\{Q_\alpha^i, Q_\beta^j\} = 2(\gamma^\mu \gamma^0)_{\alpha\beta} [(P_\mu + Z_\mu) \delta^{ij} + Z_\mu^{(ij)}] + 2\gamma_{\alpha\beta}^0 Z^{[ij]}, \tag{8}$$

where γ^0 is the charge conjugation matrix. The maximal set of 10 CC's enter as a triplet of spacetime vectors Z_μ^{ij} and a singlet $Z^{[ij]}$. The singlet CC is associated with vortices (or lumps), and corresponds to the reduction of the $(1/2, 1/2)$ charge or the 4th component of the momentum vector in $D = 4$. The triplet Z_μ^{ij} is decomposed into an R -symmetry singlet Z_μ , algebraically indistinguishable from the momentum, and a traceless symmetric combination $Z_\mu^{(ij)}$. The former is equivalent to the vectorial charge in the $\mathcal{N} = 1$ algebra, while $Z_\mu^{(ij)}$ can be reduced to a complex number and vectors specifying the orientation. We see that these are the direct reduction of the $(0,1)$ and $(1,0)$ wall charges in $D = 4$.

- $\mathcal{N} = 2$ in $D = 2$

Lorentz invariance now provides a much weaker constraint, and one can in principle consider different (p, q) superalgebras comprising $p + q = 4$ supercharges. We will mention here only the nonchiral $\mathcal{N} = (2, 2)$ case corresponding to dimensional reduction of the $\mathcal{N} = 1$ $D = 4$ algebra. The tensorial decomposition is as in (8), but with the decomposition of $D = 3$ spacetime vectors into $D = 2$ vectors and a singlet,

$$\{Q_\alpha^i, Q_\beta^j\} = 2(\gamma^\mu \gamma^0)_{\alpha\beta} [(P_\mu + Z_\mu) \delta^{ij} + Z_\mu^{(ij)}] + 2i(\gamma^5)_{\alpha\beta} (\delta^{ij} Z + Z^{(ij)}) + 2\gamma_{\alpha\beta}^0 Z^{[ij]}, \quad (9)$$

We discard all vectorial charges Z_μ^{ij} in this case for the reasons described above, and are left with two singlets $Z^{(ij)}$, which are the reduction of the domain wall charges in $D = 4$ and correspond to topological kink charges, and two further singlets Z and $Z^{[ij]}$, arising via reduction from P_2 and the vortex charge in $D = 3$.

1.7. A few words on extended supersymmetry (eight supercharges) in D=4. Complete algebraic analysis of all tensorial central charges possible in this is analogous to the previous cases and is rather straightforward. With eight supercharges the maximal number of CC's is 36. Dynamical aspect is less developed – only a modest fraction of the above 36 CC's are known to be nontrivially realized in models studied in the literature. I will limit myself to a few remarks regarding the well-established CC's. I will use a complex (holomorphic) representation of the supercharges. Then the supercharges are labeled as follows

$$Q_\alpha^f, \quad \bar{Q}_{\dot{\alpha}g}, \quad \alpha, \dot{\alpha} = 1, 2, \quad f, g = 1, 2. \quad (10)$$

On general grounds one can write

$$\begin{aligned} \{Q_\alpha^f, \bar{Q}_{\dot{\alpha}g}\} &= 2\delta_g^f P_{\alpha\dot{\alpha}} + 2(Z_g^f)_{\alpha\dot{\alpha}}, \\ \{Q_\alpha^f, Q_\beta^g\} &= Z_{\{\alpha\beta\}}^{\{fg\}} + \varepsilon_{\alpha\beta} \varepsilon^{fg} Z, \\ \{\bar{Q}_{\dot{\alpha}f}, \bar{Q}_{\dot{\beta}g}\} &= \left(\bar{Z}_{\{\dot{\alpha}\dot{\beta}\}} \right)_{\{fg\}} + \varepsilon_{\alpha\beta} \varepsilon^{fg} \bar{Z}. \end{aligned} \quad (11)$$

Here $(Z_g^f)_{\alpha\dot{\alpha}}$ are four vectorial central charges $(1/2, 1/2)$, (16 components altogether) while $Z_{\{\alpha\beta\}}^{\{fg\}}$ and the complex conjugate are $(1,0)$ and $(0,1)$ central charges. Since the matrix $Z_{\{\alpha\beta\}}^{\{fg\}}$ is symmetric with respect to f, g , there are three flavor components, while the total number of components residing in $(1,0)$ and $(0,1)$ central charges is 18. Finally, there are two scalar central charges, Z and \bar{Z} .

Dynamically the above central charges can be described as follows. The scalar CC's Z and \bar{Z} are saturated by monopoles/dyons. One vectorial central charge Z_μ (with the additional condition $P^\mu Z_\mu = 0$) is saturated [16] by Abrikosov-Nielsen-Olesen string (ANO for short) [15]. A $(1,0)$ central charge with $f = g$ is saturated by domain walls [17].

1.8. **$D = 3$: Why BPS vortices cannot appear in theories with two supercharges.** BPS vortices in 2+1 dimensions were previously considered in [18] (see also references therein). To simplify the discussion, we assume that we can choose a real representation for the superalgebra within which

$$\{Q_\alpha, Q_\beta\} = \gamma_{\alpha\beta}^\mu P_\mu + \dots, \quad (12)$$

where μ is a spacetime index. It then follows that, since the number of broken translational generators is d , there are *at least* d broken supercharges. In practice the number may be larger once we account for Lorentz invariance.

This simple argument tells us that, provided we are dealing with a 1/2-BPS soliton in a supersymmetric theory (i.e. an object localized only in space not time), the minimal matching between bosonic and fermionic zero modes in the translational sector is one-to-one.

Consider now a putative BPS vortex in a theory with minimal $\mathcal{N} = 1$ SUSY in 2+1D. Such a configuration would require a worldvolume description with two bosonic zero modes, but only one fermionic mode. This is not permitted by the argument above, and indeed no configurations of this type are known. Vortices always exhibit at least two fermionic zero modes and are thus BPS only in $\mathcal{N} = 2$ theories.

2. “MONOPOLE” CENTRAL CHARGES: ANOMALIES AND ALL THAT

In this section we will discuss the Lorentz-scalar central charges in Eq. (11) that are saturated by monopoles/dyons. They will be referred to as monopole central charges. A rather dramatic story is associated with them, a story which is not yet finished. Historically they were the first to be introduced within the framework of an extended 4D superalgebra [5, 6]. On the dynamical side, they appeared as the first example of the “topological charge \leftrightarrow central charge” relation revealed by Witten and Olive in their pioneering paper [7]. Twenty years later, the $\mathcal{N} = 2$ model where these central charges first appeared, was solved by Seiberg and Witten [20, 21], and the exact masses of the BPS-saturated monopoles/dyons found. No direct comparison with the operator expression for the central charges was carried out, however. In Ref. [22] it was noted that for the Seiberg-Witten formula to be valid, a boson-term anomaly should exist in the monopole central charges. Even before [22] a fermion-term anomaly was identified [23], which plays a crucial role [24] for the monopoles in the Higgs regime (confined monopoles). What is still lacking is a direct operator derivation of the above anomalies.

2.1. **The model.** The simplest $\mathcal{N} = 2$ model we will deal with was found [25] as early as in 1974, see also [26] where the matter multiplets – the so called hypermultiplets – first appeared.⁶ The $\mathcal{N} = 2$ model in four dimensions can be obtained from $\mathcal{N} = 1$ super-Yang-Mills theory in six dimensions.

It is instructive to consider both, the Majorana and Weyl representations. The gauge group is SU(2). In the Weyl representation one deals with two Weyl fermions, λ_α^a (gluino) and ψ_α^a (gluino's $\mathcal{N} = 2$ superpartner). Then

$$\begin{aligned} \mathcal{L} = & \frac{1}{g_0^2} \left\{ -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + \lambda^{\alpha,a} (i D_{\alpha\dot{\alpha}}) \bar{\lambda}^{\dot{\alpha},a} + \frac{1}{2} D^a D^a \right. \\ & + \psi^{\alpha,a} (i D_{\alpha\dot{\alpha}}) \bar{\psi}^{\dot{\alpha},a} + D^\mu \bar{\phi}^a D_\mu \phi^a \\ & \left. - \sqrt{2} \varepsilon_{abc} (\bar{\phi}^a \lambda^b \psi^c + \text{h.c.}) - \frac{i}{2} D^a \varepsilon_{abc} \bar{\phi}^b \phi^c \right\}, \end{aligned} \quad (13)$$

where where g_0 is the bare coupling constant, and D^a is an auxiliary field which can be eliminated by virtue of the equation of motion,

$$D^a = \frac{i}{2} \varepsilon_{abc} \bar{\phi}^b \phi^c, \quad (14)$$

while there are no F terms (i.e. they are set to zero) since we introduce no superpotential.

The flat direction of the model can be parametrized as follows:

$$\phi^3 = v, \quad \phi^1 = \phi^2 = 0. \quad (15)$$

Moreover, using the (anomalous) U(1) symmetry one can always make the vacuum expectation value v real and positive. Generally speaking, such a rotation introduces a vacuum angle θ , even if it was fine-tuned to zero in the beginning. The question of θ -induced effects is interesting by itself, but I will not consider it here, deferring the corresponding discussion till better times. Thus, I will assume $\theta = 0$ *after* setting ϕ real. For real and positive v the W -boson mass m is

$$m = \sqrt{2} v. \quad (16)$$

⁶There is a funny story about the emergence of the word “hypermultiplet” in this context. M. Sohnius recalls [27]: When a French super-marché carries not only food and drink but also car spares, garden furniture and ladies’ underwear, it becomes an hyper-marché. Correspondingly, P. Fayet called $\mathcal{N} = 2$ supersymmetry “hyper-symmetry.” Whereas that name has not stuck in general, the matter multiplet of $\mathcal{N} = 2$ supersymmetry is still called “hypermultiplet.”

At the same time, in the Majorana representation

$$\begin{aligned}\mathcal{L} &= \frac{1}{g_0^2} \left\{ -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + \frac{1}{2} \bar{\chi}_i^a i \not{D} \chi_i^a + \frac{1}{2} D^\mu A^a D_\mu A^a + \frac{1}{2} D^\mu B^a D_\mu B^a \right. \\ &= \left. +\frac{1}{2} \text{Tr}[A, B][A, B] + \frac{i}{2} \varepsilon_{ij} \text{Tr}([\bar{\chi}^i, \chi^j]A + ([\bar{\chi}^i, \gamma^5 \chi^j]B)) \right\}\end{aligned}\quad (17)$$

where χ_i ($i = 1, 2$) are two Majorana fermions, A is a scalar field and B is a pseudoscalar field, all in the adjoint representation.

At the classical level the description of monopoles does not depend on fermions at all. Let us consider static field configurations. Then, neglecting all time derivatives and setting $A_0 = 0$, one can write the Bogomolny completion of the energy functional as follows:

$$\begin{aligned}\mathcal{E} &= \int d^3x \left\{ \left[\frac{1}{\sqrt{2}g_0} F_1^{*a} + \frac{1}{g} D_1 \phi^a \right]^2 + \left[\frac{1}{\sqrt{2}g_0} F_2^{*a} + \frac{1}{g} D_2 \phi^a \right]^2 \right. \\ &+ \left. \left[\frac{1}{\sqrt{2}g_0} F_3^{*a} + \frac{1}{g} D_3 \phi^a \right]^2 \right\} - \frac{\sqrt{2}}{g_0^2} \int dS_n (\phi^a F_n^{*a}),\end{aligned}\quad (18)$$

where the last term is the surface term, written as an integral over a large sphere, and

$$F_m^* = \frac{1}{2} \varepsilon_{mnk} F_{nk}.$$

The Bogomolny equations for the monopole are

$$F_i^{*a} + \sqrt{2} D_i \phi^a = 0. \quad (19)$$

The solution to this equation is given by the famous hedgehog *ansatz* [28]

$$\phi^a(\vec{x}) = \delta^{ai} \frac{x_i}{r} F(r), \quad A_i^a(\vec{x}) = \varepsilon^{aij} \frac{x_j}{r} W(r), \quad (20)$$

where $r = \sqrt{\vec{x}^2}$ while $A_0^a = 0$. Equations (20) must be supplemented by boundary conditions at the origin (where the solution must be regular), and at the spatial infinity where $F(r) \rightarrow v$ and $W(r) \rightarrow 1$. The profile functions F and W can be found analytically, see e.g. [29].

2.2. Dimension of the BPS representations. As was first noted by Montonen and Olive [34], all states in $\mathcal{N} = 2$ model – W bosons and monopoles alike – are BPS saturated. This results in the fact that supermultiplets of this model are short. Regular (long) supermultiplet would contain $2^{2\mathcal{N}} = 16$ helicity states, while the short ones contain $2^{\mathcal{N}} = 4$ helicity states – two bosonic and two fermionic. This is in full accord with the fact that the number of the fermion zero mode on the given monopole solution is four, resulting in dim-4 representation of the supersymmetry

algebra. If we combine particles and antiparticles together, as is customary in field theory, we will have one Dirac spinor on the fermion side of the supermultiplet in both cases, W -bosons/monopoles.

2.3. Supercurrents. The model, being $\mathcal{N} = 2$, possesses two conserved supercurrents,

$$\begin{aligned} J_{\alpha\beta\dot{\beta}}^I &= \frac{1}{g_0^2} \left\{ 2i F_{\beta\alpha}^a \bar{\lambda}_{\dot{\beta}}^a - 6\varepsilon_{\beta\alpha} D^a \bar{\lambda}_{\dot{\beta}}^a + 2\sqrt{2} (D_{\alpha\dot{\beta}} \bar{\phi}^a) \psi_{\beta}^a \right\}, \\ J_{\alpha\beta\dot{\beta}}^{II} &= \frac{1}{g_0^2} \left\{ 2i F_{\beta\alpha}^a \bar{\psi}_{\dot{\beta}}^a - 6\varepsilon_{\beta\alpha} D^a \bar{\psi}_{\dot{\beta}}^a - 2\sqrt{2} (D_{\alpha\dot{\beta}} \bar{\phi}^a) \lambda_{\beta}^a \right\}. \end{aligned} \quad (21)$$

The commutator of the corresponding supercharges is

$$\begin{aligned} \{Q_{\alpha}^I, Q_{\beta}^{II}\} &= -\frac{2\sqrt{2}i}{g_0^2} \int d^3x \operatorname{div} \left(\bar{\phi}^a (\vec{E}^a - i\vec{B}^a) \right) \\ &= -\frac{2\sqrt{2}i}{g_0^2} \int dS_n (\bar{\phi}^a (E_n^a - iB_n^a)). \end{aligned} \quad (22)$$

Everything is perfectly okay at the classical level. A crucial feature that I must stress is the chiral structure of the classical central charge in Eq. (22). The classical U(1) current argument tells us that the anticommutator $\{Q_{\alpha}^I, Q_{\beta}^{II}\}$ must be proportional to $\bar{\phi}$ rather than ϕ ; then, the particular structure of the supercurrents (22) implies that the color-electric and color-magnetic fields enter in the combination $E_n^a - iB_n^a$.

2.4. Coupling constant renormalization. A straightforward analysis seems to show⁷ that the impact of renormalization reduces to the replacement of the bare coupling g_0^{-2} in Eq. (22) by the (renormalized) effective coupling g^{-2} normalized at the lowest relevant mass scale in the theory, i.e. at the scale v . Then we arrive at

$$\{Q_{\alpha}^I, Q_{\beta}^{II}\} = -\frac{2\sqrt{2}i}{g^2} \int dS_n (\bar{\phi}^a (E_n^a - iB_n^a)). \quad (23)$$

If one substitutes the color-electric and color-magnetic fields generated by the electric and magnetic charge, respectively, one arrives at the following expression for the central charge (and, correspondingly, the mass of the BPS state):

$$M = \left| \sqrt{2}v \left(1 - \frac{4\pi i}{g^2} \right) \right|. \quad (24)$$

⁷Quantum corrections in the mass of the BPS saturated monopoles were first discussed in Refs. [30, 31, 32] two decades ago.

Let me parenthetically note that the general formula is

$$M = \left| \sqrt{2}v \left(n_e - \frac{4\pi i}{g^2} n_m \right) \right|, \quad (25)$$

where $n_{e,m}$ are integer electric and magnetic numbers, but we will consider here only the particular case when either $n_e = 1$ or $n_m = 0, 1$. It must be noted, with satisfaction, that for $n_m = 0$ and $n_e = 1$ we get the correct W -boson mass.

So far, everything seems perfectly cloudless. An indication that a problem exists came from comparison of the result quoted in Eq. (24) with the exact solution found in Ref. [20] in the limit of large v when all nonperturbative effects are neglected. Such a comparison could have been made immediately after publication of Ref. [20], but for reasons beyond my comprehension (and I must admit my own guilt too) it was not carried out until recently, see [22].

2.5. Detecting an anomaly. The masses of the BPS-saturated states (W bosons and monopoles) in the Seiberg-Witten exact solution can be presented by the formula

$$M = \sqrt{2} \left| a \left(n_e - \frac{a_D}{a} n_m \right) \right|, \quad (26)$$

where

$$a_D = i a \left(\frac{4\pi}{g_0^2} - \frac{2}{\pi} \ln \frac{M_0}{a} \right), \quad (27)$$

while the renormalized coupling constant is defined in terms of the ultraviolet parameters as follows:

$$\frac{\partial a_D}{\partial a} \equiv \frac{4\pi i}{g^2}. \quad (28)$$

Because of the $a \ln a$ dependence, $\partial a_D / \partial a$ differs from a_D / a by a constant (nonlogarithmic) term, namely,

$$\frac{a_D}{a} = i \left(\frac{4\pi}{g^2} - \frac{2}{\pi} \right). \quad (29)$$

Combining Eq. (26) and (29) we get

$$M = \sqrt{2} \left| a \left(n_e - i \left(\frac{4\pi}{g^2} - \frac{2}{\pi} \right) n_m \right) \right|, \quad (30)$$

This does not match Eq. (25) in the nonlogarithmic part (i.e. the part of the n_m term with no g^2 factor). Since the relative weight of the electric and magnetic parts in Eq. (23) is fixed to be $\sim g^2$, the presence of the above nonlogarithmic term implies that, in fact, the chiral structure $E_n^a - i B_n^a$ obtained at the canonic commutator level cannot be maintained once quantum corrections are switched on. This is a quantum anomaly.

2.6. Getting the anomaly. First, I will present my argument and then try to summarize that of Rebhan *et al.* [22], although I must admit that so far I failed to make myself comfortable with the latter paper. (I am afraid, there is something not quite kosher there; in any case, further analysis seems to be needed).

Our starting point will be the (superconformal) anomaly [33] in the supercurrent (21), namely,

$$\left(\varepsilon^{\alpha\beta} J_{\alpha\beta\dot{\beta}}^I\right)_{\text{anom}} = \frac{N}{8\pi^2} \bar{F}_{\dot{\alpha}\beta}^a \bar{\lambda}^{a,\dot{\alpha}} \quad (31)$$

for $SU(N)$. Please, note the occurrence of the opposite-chirality field strength tensor $\bar{F}_{\dot{\alpha}\beta}^a$. At the classical level the current $J_{\alpha\beta\dot{\beta}}^I$ contains $F_{\alpha\beta}^a$ (which eventually leads to $E_n^a - i B_n^a$ in the anticommutator (23)) rather than $\bar{F}_{\dot{\alpha}\beta}^a$. The fact that at the quantum level $\bar{F}_{\dot{\alpha}\beta}^a$ pops up means that the anticommutator (23) does have an anomaly – an $\mathcal{N} = 2$ relative of the superconformal anomaly – which gives rise to $E_n^a + i B_n^a$, a term of the opposite chiral structure. A rather straightforward calculation then gives

$$\{Q_\alpha^I, Q_\beta^{II}\}_{\text{anom}} = -2\sqrt{2}i \frac{1}{4\pi^2} \int dS_n (\bar{\phi}^a (E_n^a + i B_n^a)) , \quad (32)$$

to be compared with Eq. (22). (In the $SU(N)$ we would have $N/(8\pi^2)$ instead of $1/(4\pi^2)$ in Eq. (32).) Adding the canonic and the anomalous terms in $\{Q_\alpha^I, Q_\beta^{II}\}$ together we see that the fluxes generated by color-electric and color-magnetic terms are now shifted, untied from each other, by a nonlogarithmic term in the magnetic part. Normalizing to the electric term, $M_W = \sqrt{2}v$, we get for the magnetic term

$$M_M = \sqrt{2}v \left(\frac{4\pi}{g^2} - \frac{2}{\pi} \right) , \quad (33)$$

as it is necessary for the consistency with the exact Seiberg-Witten solution.

A few words about the analysis of of Rebhan *et al.* [22]. These authors did not aim at establishing the operator form of the anomaly. Instead, they started from the assumption that central charges relevant to the monopole problem in four dimensions can be viewed as a dimensional reduction of the Golfand-Likhtman superalgebra (1). Then they calculated the matrix element of the energy-momentum tensor (more exactly, its fermion part) in $4+\epsilon$ dimensions in the monopole background field. Upon analytic continuation to $\epsilon \rightarrow 0$ they find a finite nonlogarithmic term consistent with (33) which is interpreted as an anomaly.

That the Golfand-Likhtman superalgebra (1) generates central charges upon dimensional reduction is known for a long time (see e.g. the book [8]). A crucial question is whether all relevant supercharges can be obtained through this procedure. In the problem at hand the answer is negative.

To illustrate this assertion let us consider $\mathcal{N} = 1$ Yang-Mills theory in $D = 6$. As well-known, dimensional reduction of this theory to $D = 4$ gives rise to four-dimensional $\mathcal{N} = 2$ model we deal with here. Assume that the six-dimensional

superalgebra has the form (1) with

$$P_\mu = \int \theta_{\mu 0}(x) d^5 x \quad (34)$$

(here $\theta_{\mu 0}$ is the six-dimensional energy-momentum tensor), dimensionally reduced to $D = 4$. Then ask whether the result for $\mu = 4, 5$ can reduce to Eq. (22). The answer seems to be negative. Let me explain why.

First note that the Weyl spinor in six dimensions has four (complex) components, $\Psi = \{\Psi_{1,2,3,4}\}$ while σ^μ matrices can be chosen as follows:

$$\sigma^\mu = \{1, \gamma^0 \gamma^1, \gamma^0 \gamma^2, \gamma^0 \gamma^3, \gamma^0, i \gamma^0 \gamma^5\}, \quad (35)$$

so that all spatial matrices are Hermitean and anticommuting. (Here $\gamma^{\mu,5}$ are the Dirac matrices.) Then, suppressing the color indices, one can write

$$\mathcal{L}_{D=6} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \Psi^\dagger \sigma^\mu (i D_\mu) \Psi, \quad (36)$$

where I have also set $g^2 = 1$ to ease the notation. The gauge coupling constant can be easily restored at the very end. In the six-dimensional language this Lagrangean in $\mathcal{N} = 1$, i.e. it has eight supercharges (see Table 1). In four dimensions this theory is $\mathcal{N} = 2$. The dimensional reduction is carried out in a standard way, namely, $x_{4,5}$ are compactified, and only zero modes in $x_{4,5}$ are retained. In practical terms this means that we just drop the $x_{4,5}$ dependence. Given our choice of the σ^μ matrices, from the 4D perspective, A_4 is a scalar field while A_5 is pseudoscalar. These two fields can be combined into a complex field

$$\phi = \frac{A_4 + i A_5}{\sqrt{2}}. \quad (37)$$

We will also need the expression for the energy-momentum tensor, which has the following canonic form:

$$\begin{aligned} \theta^{\mu\nu} &= F^{\mu\alpha} F_{\alpha}{}^\nu - \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} \\ &+ \frac{i}{4} \left\{ \Psi^\dagger \sigma^\mu D^\nu \Psi + \Psi^\dagger \sigma^\nu D^\mu \Psi - \Psi^\dagger \sigma^\mu \overleftarrow{D}^\nu \Psi - \Psi^\dagger \sigma^\nu \overleftarrow{D}^\mu \Psi \right\}. \end{aligned} \quad (38)$$

Upon dimensional reduction to $D = 4$, combining this expression with equations of motion, one readily gets

$$\begin{aligned} \theta^{04} &= -\text{div} \left(A^4 \vec{E} \right) - \frac{i}{4} \text{div} \left(\Psi^\dagger \vec{\gamma} \Psi \right), \\ \theta^{05} &= -\text{div} \left(A^5 \vec{E} \right) + \frac{1}{4} \text{div} \left(\Psi^\dagger \vec{\gamma} \gamma^5 \Psi \right). \end{aligned} \quad (39)$$

Here we omitted terms vanishing by virtue of equations of motion. Quick inspection shows that $\int d^3x \theta^{04(5)}$ cannot represent the central charge in full, as it is obvious from Eq. (22) that at the classical level the color-electric and color-magnetic fields enter in a unified chiral combination $E_n^a - i B_n^a$, while the dimensional reduction of the energy-momentum tensor gives rise only to the color-electric field. How does the color-magnetic field emerge?

This is only possible if the algebra (1) gets a central extension already at 6D-level. It is not difficult to infer a general structure of this central extension. Namely,

$$\begin{aligned} \{\bar{Q}_\alpha Q_\beta\} &= 2 (\sigma^A)_{\alpha\beta} \int dx \{ \theta_{0A} \\ &+ \varepsilon_{0ABCDE} \partial^B \left(\frac{i}{24} \Psi^\dagger \sigma^C \sigma^D \sigma^E \Psi + A^C \partial^D A^E + \dots \right) \}, \end{aligned} \quad (40)$$

where the ellipses stand for the non-Abelian part of the gauge-boson term. Upon reduction to four dimensions the fermion part in the second line in Eq. (40) cancels that in Eq. (39) – at the classical level there are no fermion terms in the anti-commutator of the supercharges, as is clearly seen in Eq. (22). The boson part generates missing terms with the color-magnetic fields which complete the boson part of Eq. (39) making it compatible with (22).

A question which immediately comes to my mind is whether one can use dimensional reduction (from $4 + \epsilon$) to obtain the anomalous part of the central charge. Since the second line in Eq. (40) contains ϵ , its continuation to $4 + \epsilon$ is problematic, to put it mildly. It is not clear to me at all how to treat it in $4 + \epsilon$. On the other hand, the fermion terms in Eq. (39) can be trivially extended to $4 + \epsilon$. If one subtracts the part at $D = 4$ (which is presumably cancelled (?) by (40), the difference is, naturally, proportional to ϵ . A properly defined gauge-field-background loop with the fermion vertices from Eq. (39) (which will also require specification of γ^5 in $4 + \epsilon$), being divergent, provides $1/\epsilon$ times, presumably, $\text{div} \left(A^5 \vec{E} \right)$ or $\text{div} \left(A^4 \vec{B} \right)$. The product is ϵ independent and finite at $\epsilon = 0$ which certainly smells of anomaly. At least operationally, this is what happens in the calculation of Ref. [22] which, as was mentioned above, proves to be compatible with the exact Seiberg-Witten formula. A task for the future is to work out a fully transparent operator interpretation of the procedure, along the lines discussed above. The same mechanism which is responsible for the generation of the bosonic anomaly destroys, at one-loop level, the cancellation of the bifermion terms which took place at the tree level. This gives rise to the fermion part of the central charge anomaly. In fact, the occurrence of such anomalous terms had been inferred previously [17, 24].

2.7. Bifermion part of the anomaly. The story begins with the discovery [10] of the gluino condensate term in the domain-wall central charge in $\mathcal{N} = 1$ supersymmetric gluodynamics. The fact that it should have $\mathcal{N} = 2$ superpartners was first mentioned in [17]. It is not difficult to see that in the general case, in the model under consideration

$$\mathcal{Z}^{fg} \sim \frac{N}{16\pi^2} \lambda_\alpha^f \lambda_\beta^g, \quad (41)$$

where f and g are “subflavor” indices (i.e. two Weyl spinors, λ and ψ in Eq. (13)). The same indices f and g label the supercharges, see Eq. (21). There are two options: one can antisymmetrize with respect to α and β and symmetrize with respect to f and g , and *vice versa*. In the first case we get the domain-wall central charges, while in the latter case obviously arrive at an anomaly in the monopole central charge. The origin is common. In Ref. [24] it was established that, for the SU(2) model,

$$\begin{aligned} \{Q_\alpha^f Q_\beta^g\} &= \varepsilon_{\alpha\beta} \varepsilon^{fg} 2 \int d^3x \zeta^0(x), \\ \zeta^0 &= \frac{1}{2} \varepsilon^{0\nu\rho\sigma} \frac{1}{8\sqrt{2}\pi^2} \partial_\nu (\lambda_{f\alpha}^a (\sigma_\rho)^{\alpha\dot{\alpha}} (\bar{\sigma}_\sigma)_{\dot{\alpha}\beta} \lambda^{af\beta}). \end{aligned} \quad (42)$$

No direct contact with the consideration of Ref. [22] has been established so far.

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